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Velocity measurements in axisymmetric jets of dilute polymer solutions

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Experiments are described in which the velocity profiles of axisymmetric jets are measured in water and in dilute solutions of water-soluble polymers. The ambient fluid and that entering through the nozzle were homogeneous. The similarity characteristics of velocity profiles including a linear variation of centre-line velocity with the inverse of the distance from the nozzle were unaltered by these substances. On adding large polyethylene oxide molecules to the system the jet spread at an increased angle, possibly due to the entrainment rate being enhanced by an increase in turbulent elasticity for polyethylene oxide solutions. It is also postulated that additives which had no effect on the velocity were not large enough to influence the small microscale eddies in which the energy to activate an entrainment cycle is stored.

1. Introduction

Some considerable interest has been shown in the effect of minute quantities of polymeric solutes in the turbulent flow structure of the solvent. One of the most remarkable effects is the reduction in pressure drop for a given flow rate in turbulent pipe flow first noted by Toms (1948). This problem has been considered in several recent papers, Metzner & Park (1964), Fabula (1965), Astarita & Nicodemo (1966) and Meyer (1966). Drag reduction also occurs in Couette flow (Hoyt & Fabula 1964; Shin 1965). It appears that high-molecular-weight linear polymers are better drag-reducing agents than branched polymers (Hoyt & Fabula 1964), and that for concentrated solutions better fractional reduction of drag coefficient was obtained for solutions with larger ratios of elastic to normal stresses (Metzner & Park 1964).

The effect of additives on free turbulent flows has not received so much attention to date. Gadd (1965) published photographs of jets in water and in polyethylene oxide, dye being added to the fluid entering the jet. Near the nozzle the jet is laminar but on transition to turbulence, the dye in the polymer solution does not distribute evenly like its water counterpart but breaks up into a series of mushroom-shaped concentrations of dye connected by narrow filaments of coloured fluid. In the same paper some qualitative results on jet velocity profiles are also given. As a result of this work it seemed that a further investigation of

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free turbulence in dilute polymer solutions would be likely to throw further light upon the phenomenon. In the present paper the mean velocity flow field in the region of the jet near the orifice is examined.

2. Experimental

A flow diagram of the apparatus is given in figure 1 (*a*). The jet (C) pushed water vertically upward into a tank (A) which was made of Perspex, 40 × 40 cm at the base and 75 cm in depth. Water flowed out of the tank over a broad-crested weir (B) on three sides of the tank and down into a collecting tank (G) of approximately 20 l. capacity. This tank primed the centrifugal pump (F) which pumped liquid through the rotometer (E) into a settling tank (D) made of brass 30 cm in diameter and 60 cm in height. From the settling tank fluid was forced under pressure through the jet. Rotometer (E) was metric size 35 and capable of reading flow rates of 25–250 c.c./sec. A pressure gauge inserted between F and E showed that

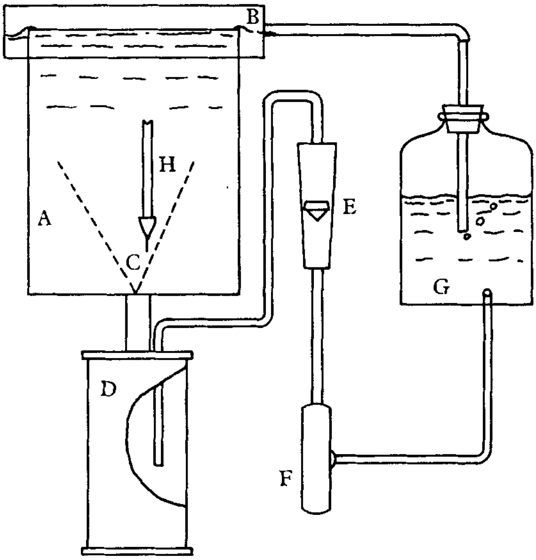

FIGURE 1 (*a*). Diagram of apparatus.

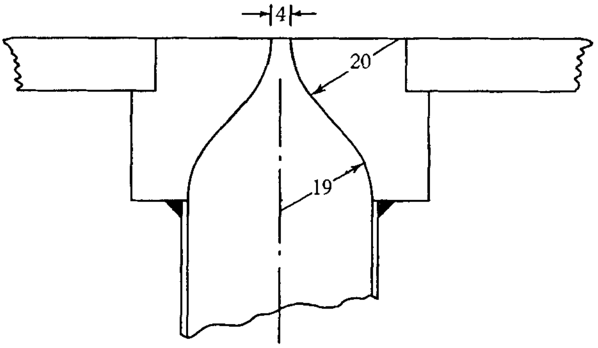

FIGURE 1 (*b*). Details of jet (dimensions in mm).

the maximum pressure in the pipes was 20 lb./in.^{2} gauge and it was found that $\frac{7}{8}$ in. inner diameter PVC tubing was sufficiently robust to carry the water from one part of the apparatus to the other. Details of the nozzle of the jet (C) are shown in figure 1 (*b*). This was made by shaping a large tool to the profile of the nozzle and using it to turn a piece of brass on a lathe. With this nozzle and pump, jet Reynolds numbers ($\mathrm{Re} = 4Q/\pi\nu D$) of 63,000–34,000 were obtained. (Here Q is the flow rate through which the nozzle, D is the nozzle diameter and ν the kinematic viscosity.)

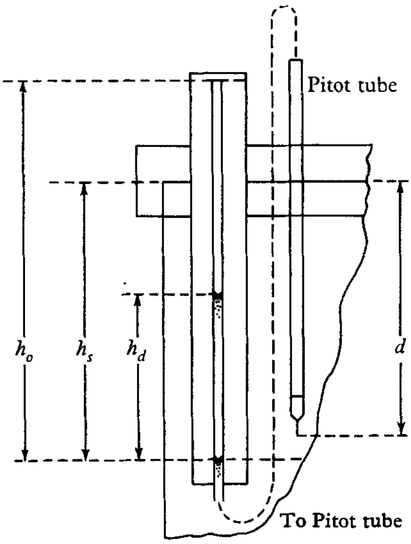

FIGURE 2. Use of closed-end manometer.

The velocity of the flow parallel to the axis of the jet was measured with a Pitot tube 8 cm long and 1 mm in outside diameter connected by a specially made streamlined support to a length of 1 in. pipe shown in figure 1 (*a*) by H. The stagnation pressure at the end of the Pitot tube was measured using a closed-end manometer, shown in figure 2. h_a is the atmospheric pressure in cm of water. With the pump switched off, the static reading on the manometer scale is h_s cm below the weir of the jet tank. When the fluid is in motion the increase in stagnation pressure forces the fluid to rise in the manometer by h_d cm. The end of the manometer tube is h_0 cm above the position of the static reading.

When stagnant the pressure of the air in the manometer is $h_a + h_s$. When the jet is flowing the pressure in the gas is given by Boyle's Law as $(h_a + h_s)\, h_0/(h_0 - h_d)$. The increase in pressure at the Pitot tube is

$$h_d + h_0(h_a + h_s)/(h_0 - h_d) - h_a - h_s = u^2/2g.$$

Hence

$$u = [2gh_d\{1 + (h_a - h_s)/(h_0 - h_d)\}]^{\frac{1}{2}}. \qquad (1)$$

Usually $h_a \gg h_s$ and $h_0 \gg h_d$; (1) becomes

$$u \sim [2gh_d(1 + h_a/h_0)]^{\frac{1}{2}}. \qquad (2)$$

With h_0 infinite we have the expression for the velocity as measured in an open-end manometer. The closed-end manometer thus has its accuracy for small velocities decreased by a factor of $[1+(h_a/h_0)]^{\frac{1}{2}}$ when compared with the open-end manometer. In these experiments $h_a \sim 1000$ cm, $h_0 \sim 140$ cm giving $[1+(h_a/h_0)]^{\frac{1}{2}} \sim 3$. Though less accurate than an open-end manometer the close-end variety is capable of measuring large stagnation pressures where otherwise a mercury manometer would have to be used and so is far more versatile.

The Pitot tube could be moved in a horizontal plane by use of a lathe traverse. This has two worm screws perpendicular to each other making it possible to place the Pitot tube anywhere within a 20 cm square. The position of the end of the Pitot tube from the centre of the nozzle was measured using two travelling microscopes with telescopic attachments, one for vertical the other for horizontal measurements. Alignment of the vertical travelling microscope was easily accomplished using a spirit level. The alignment of the horizontal one was more complicated, since it was necessary to get it parallel to one of the worm screws of the lathe traverse. A trial and error method was used. When the telescope was aligned correctly, the Pitot tube did not 'drift' in the scale in the eyepiece when it was moved the full 20 cm in the direction perpendicular to the cross-bar scale on the travelling microscope.

The polymers were dissolved directly into the tank. These were first made into a slurry by mixing them in alcohol, this preventing the formation of large lumps of partially dissolved polymers which occur when the dry powder is added to water. The solutions were left overnight to ensure proper mixing. Intrinsic viscosity measurements were made in an Ostwald viscometer and the molecular weight M_w calculated from the formula:

$$\text{intrinsic viscosity } [\eta] = 1{\cdot}25 \times 10^{-4} M_w^{0{\cdot}78}.$$

3. Results

The following quantities are used in this discussion:

(i) U the velocity in a vertical direction (x) at a point x from the nozzle end at a radial distance (r) from the centre-line;

(ii) U_M, the maximum velocity for a given x;

(iii) $r_{\frac{1}{2}}$, the radius at which $U = \frac{1}{2}U_M$ for a fixed x;

(iv) D, the diameter of the nozzle; Q, the discharge; and $U_0 = 4Q/\pi D^2$ the mean velocity of fluid at the jet nozzle.

When measuring velocity profiles the Pitot tube was taken to one side of the point of maximum velocity and traversed across it so that the centre-line could be detected as accurately as possible. Centre-line velocities U_M were found by finding the point of maximum velocity by a trial and error adjustment of the lathe traverse in two perpendicular directions. The apparatus was tested by carrying out a run in water (figures 3, 4) at a nozzle Reynolds number of 64,700. Figure 3 gives data obtained for velocity profiles plotted with U/U_M as ordinate and $r/r_{\frac{1}{2}}$ as abscissa. The results for the four traverses are in good agreement with Tollmien's solution for an axisymmetric source (Abramovich 1963, p. 76). The centre-line velocities are given in figure 4; three separate runs are shown over a period of two months. These show the expected relationship $U_0/U_M = 2ax/0{\cdot}96D$ (Abramovich 1963, pp. 82–5). Figure 4 indicates a value of 0·069 for the para-

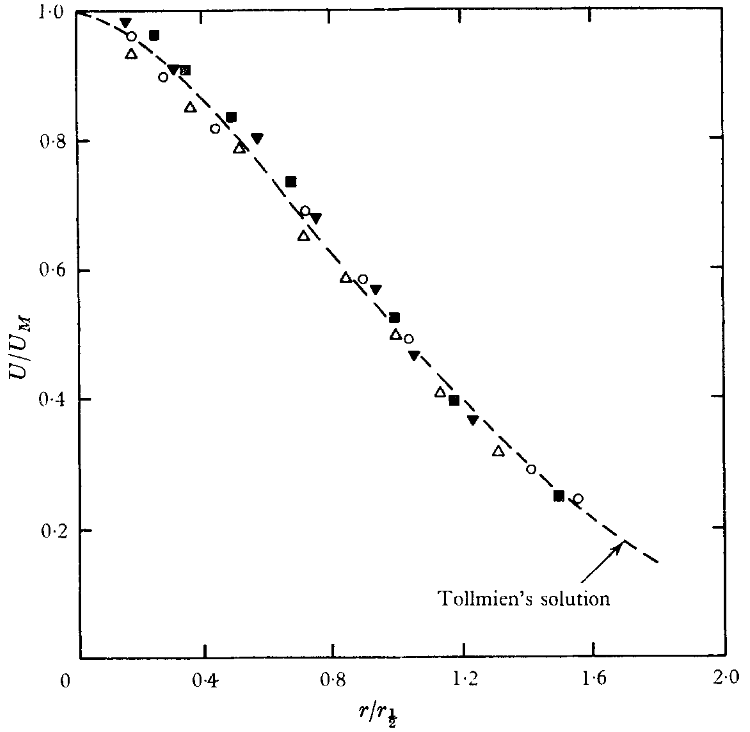

FIGURE 3. Velocity profiles for water. ■, x/D = 14·9; ○, x/D = 10·7; ▼, x/D = 21·1; △, x/D = 24·7.

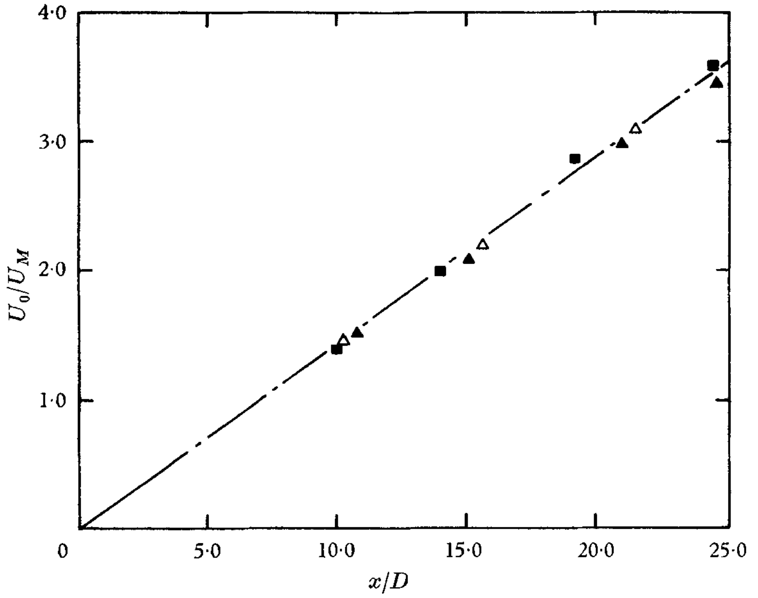

FIGURE 4. Centre-line velocities for water jet. Separate symbols used for different reproducibility runs.

meter a. Abramovich quotes values of 0·066 for experiments carried out in Göttingen and 0·070 for Zimm's experiments. The present value of 0·069 is also close to the value of 0·073 for experiments by Corrsin as reported by Rossler & Bankoff (1963).

Figure 5 gives centre-line velocities for two polymers examined in this investigation, Guar Gum and Hydroxyethyl Cellulose (HEC). Both substances are based on chains of cellulose rings but have different radicals attached to the rings. Guar Gum formed a milky suspension in water at both concentrations used. Measurements were taken at five different nozzle Reynolds numbers between 34,000 and 64,000. The HEC was grade 250HH of 'Natrosol' kindly supplied by the Hercules Powder Company. This polymer has a greater affinity for water than Guar Gum and forms clear solutions at a concentration of 200 p.p.m. The centre-line velocities were made at three different Reynolds numbers in the range 34,000 to 64,000 and again no change in centre-line velocity was noted.

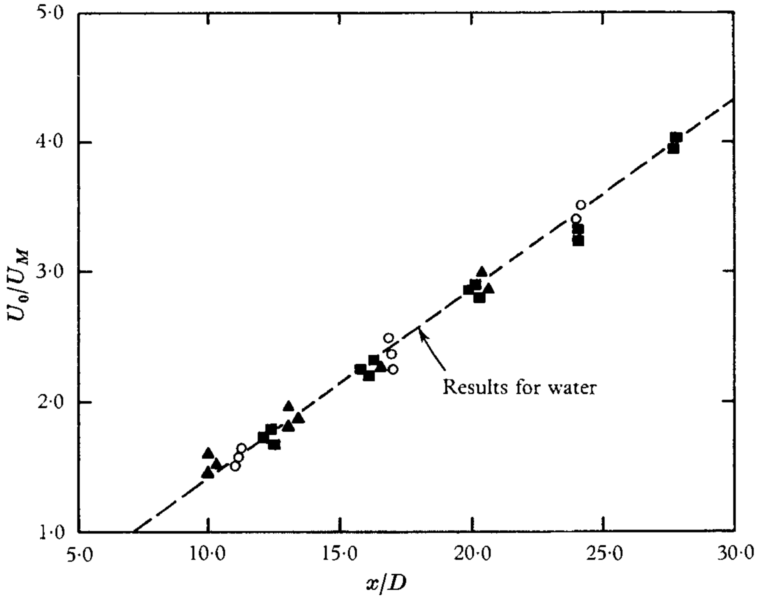

FIGURE 5. Centre-line velocities for Guar Gum and Hydroxyethyl Cellulose. ■, Guar Gum 75 p.p.m.; ○, Guar Gum 150 p.p.m.; ▲, HEC 200 p.p.m.

The third polymer used was 'P301 Polyox' which was kindly provided by Union Carbide Ltd. This is a comparatively well-studied polymer having a molecular weight of approximately $3{\cdot}6 \times 10^6$ and an intrinsic viscosity $[\eta]$ of 16·0 dL/g. It is a long unbranched chain of polyethene oxide $[-CH_2CH_2O-]_N$ and is very susceptible to mechanical degradation so that the polymer chains are reduced in size whenever any large stress is applied to the molecules. A certain amount of degradation occurred during mixing. The solutions obtained had a viscosity close to that of water, they were clear, had a slightly soapy consistency and a tendency to drip down in long thin filaments. When stirred they did not exhibit any noticeable viscoelastic property such as recoil or the Weissenberg effect. As is shown in figure 6, at a molecular weight of $1{\cdot}9 \times 10^6$ it was impossible to obtain a reasonable straight line for (U_0/U_M) vs (x/D). The centre-line measurements in this case were made from the bottom upwards. At low x/D the results lie below the dotted line A because the polymer has not yet gone into solution properly. At an x/D of 20 the results lie above A and mixing is complete. By the time measurements were taken at $x/D = 22$ degradation had set in and the results move closer to A. When the molecular weight had dropped to $1{\cdot}2 \times 10^6$ the degradation is much slower and the centre-line velocity variation can be measured as a straight line on a (U_0/U_M) vs (x/D) plot. Further degradation takes place down to $1{\cdot}1 \times 10^6$ and another straight line is obtained on the graph. At this point the rate of degradation had slowed down a great deal.

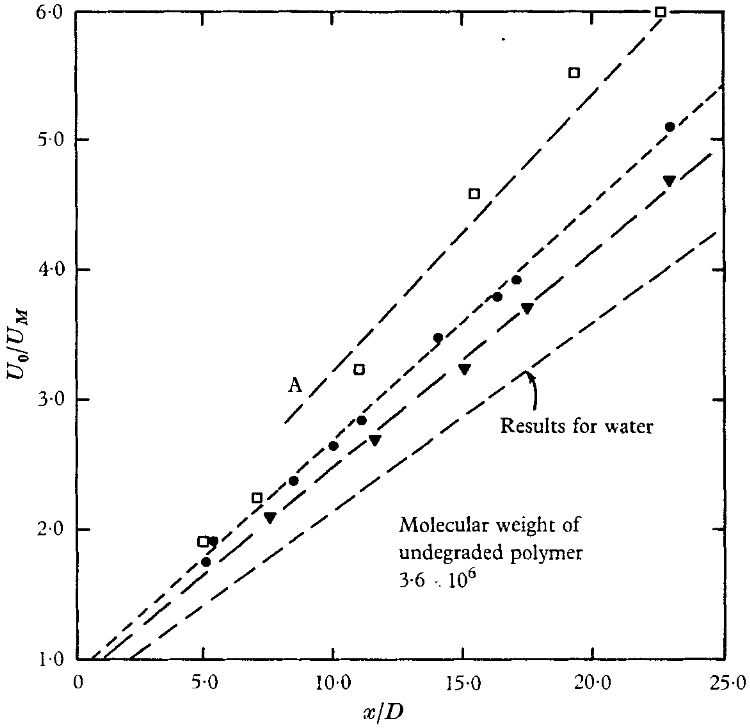

FIGURE 6. Centre-line velocities for 50 p.p.m. 'Polyox' solution. □, M.W. $1{\cdot}9 \times 10^6$; ●, M.W. $1{\cdot}2 \times 10^6$; ▼, M.W. $1{\cdot}1 \times 10^6$.

Velocity profile data for this solution are shown in figure 7. The use of (U/U_M) and $(r/r_{\frac{1}{2}})$ as co-ordinates brings the data on to one experimental curve which differs slightly from Tollmein's solution, though possibly not by a significant amount. Since the similarity solution seems to exist then the momentum of the jet at any cross-section will be $kU_M^2 r_{\frac{1}{2}}^2$, where k is a geometric constant computed to be 2·3 by numerical integration of the results given in figure 7. The product of U_M and $r_{\frac{1}{2}}$ was found to be approximately constant with a scatter of ± 4%. The mean result for momentum from these measurements is $3{\cdot}1 \times 10^5$ cm⁴/sec² which compares closely with the actual initial momentum through the jet of $3{\cdot}5 \times 10^5$ cm⁴/sec. We deduce that momentum is conserved within the 50 p.p.m. 'Polyox' jet. This also serves as a check on the validity of the velocities measured using a

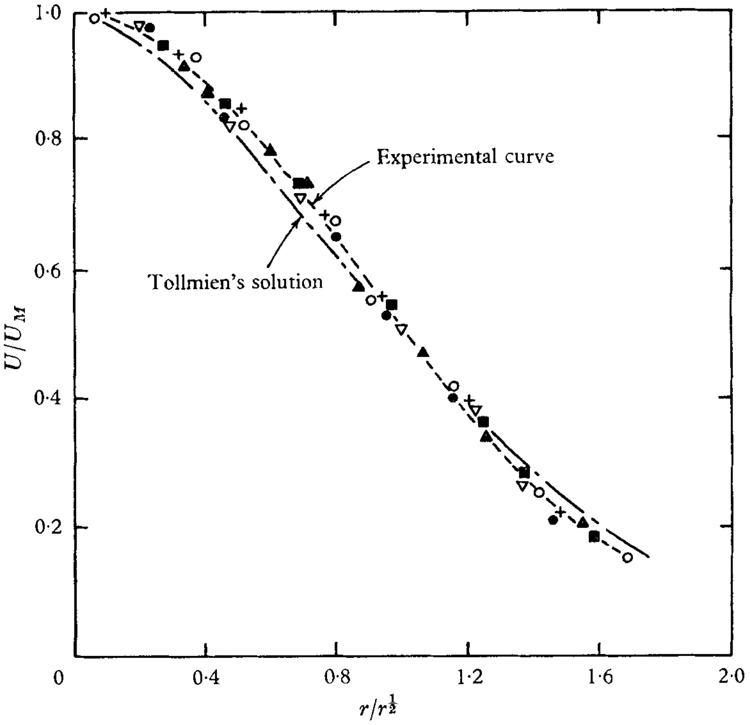

FIGURE 7. Velocity profiles for 50 p.p.m. solution of 'Polyox'. ▲, x/D = 9·57; ○, x/D = 13·3; ■, x/D = 16·9; ▽, x/D = 20·4; +, x/D = 23·9; ●, x/D = 30·2.

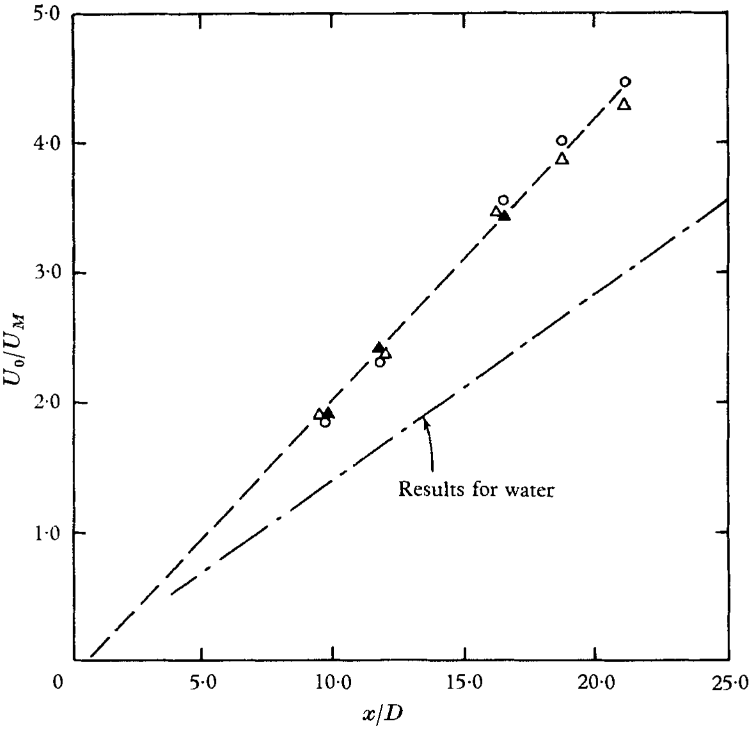

FIGURE 8. Centre-line velocities in 100 p.p.m. 'Polyox' solution. △, Re = 64,700; ○, Re = 48,300; ▲, Re = 34,700.

Pitot tube in this solution, in view of the doubts that exist about using Pitot tubes for velocity profile measurements of turbulent flow in polymer solutions of concentrations 500 to 2000 p.p.m. (Astarita & Nicodemo 1966).

Finally, on figure 8 centre-line velocities for a 100 p.p.m. Polyox jet are shown. The molecular weight is about $3{\cdot}0 \times 10^6$. Three Reynolds numbers were used in these measurements, but all fall on the same straight line showing that the equation $U_0/U_M = 2ax/0{\cdot}96D$ holds for this solution. The value of a given by figure 8 is approximately 0·099.

The case for the validity of these measurements is not so clear-cut as for the 50 p.p.m. solution since it was not possible to make a momentum traverse. However the way U_0/U_M scales so well on figure 8 at three different *Re* in accordance with the theory of the pure solvent jet argues for the existence of a linear spread. Further work is still needed on the interpretation of Pitot tube measurements in polymer solutions.

4. Conclusions

When a polymer is added to the system the velocity at a given point may be reduced, this reduction increasing with molecular weight. Apart from this the similarity properties of the jet are unaltered; momentum is conserved, similar velocity profiles are obtained on $\{(U/U_M), (r/r_{\frac{1}{2}})\}$ diagrams and the relation $U_0/U_M = 2ax/0{\cdot}96D$ describes the centre-line velocity profile along the axis providing the nozzle Reynolds number is high enough. Two questions arise: why is the parameter a increased on adding polymer and why is this effect restricted to certain polymers? The first question is interesting for the following reason. Abramovich (1963) shows that the parameter a is increased by residual amounts of turbulence in the nozzle. This is difficult to eliminate completely, but if one thinks of 'Polyox' as a turbulence suppressor then it is not unreasonable to expect reduced residual turbulence in the nozzle and thus a *lower a* not a *higher* one as is obtained in the present experiments.

A possible explanation of this apparent paradox comes from consideration of the entrainment process in free turbulent flow. Townsend (1966) supposes that the regions of high vorticity of the jet adjacent to the comparatively still fluid can have a certain elasticity (G) associated with them and the concept may be used in the present work. The elasticity increases with the turbulent intensity of the fluid and its effect is to store up energy within the turbulent microscales to activate the entrainment of low vorticity fluid. In the presence of polymer molecules this elasticity may be changed.

The flow rate of fluid along the axis of the jet (F) is given by

$$F = k\rho U_0^2 D^2/U_M, \tag{3}$$

where ρ is the density of the fluid and k is used in the subsequent discussion as a numerical constant. Since $U_0/U_M = ka(x/D)$ the rate of increase in the flow rate along the jet axis is

$$dF/dx = k\rho a U_0 D; \tag{4}$$

dF/dx is also equal to the rate of entrainment of ambient fluid which is a function of G, U_M and $r_{\frac{1}{2}}$, the latter two quantities being velocity and length scales associated with turbulent eddies responsible for the entrainment process. U_M and $r_{\frac{1}{2}}$ are related through conservation of momentum by

$$U_M r_{\frac{1}{2}} = kU_0 D. \tag{5}$$

Consequently from dimensional arguments

$$\begin{aligned} dF/dx &\sim kGr_{\frac{1}{2}}/U_M \\ &\sim kGU_0 D/U_M^2. \end{aligned} \tag{6}$$

Combining (4) and (6) G may be written

$$G = kp a U_M^2; \tag{7}$$

due to the increase in the parameter a for a 100 p.p.m. Polyox solution, G is 40 % greater in this Polyox solution for all given U_M. The essential conclusions that follow from this simple model are that vestigial turbulence in the nozzle and addition of some visco-elastic polymer molecules increase the elasticity associated with the turbulent fluid and also increase the angle at which the jet spreads.

It is still necessary to explain why only Polyox was found to have any effect on the jet. This is possibly due to the size of the polymer molecules. If these are not large enough to effect the small eddies in which the turbulent energy for entrainment is stored then they are unlikely to affect the flow. The dimensions of the molecules or molecule groups is probably an important parameter. The size of a molecule in random coiling is given as $L\sqrt{n}$ (Gordon 1963) where L is the length of a monomer and n is the number of monomers in a chain. Data for Guar Gum (Hoyt & Fabula 1964) indicate that $L \sim 8$ Å, and n is about 1000; for Polyox $L \sim 3$ Å, $n = 250{,}000$. The size of a Polyox molecule is a sphere roughly 1500 Å in diameter whereas Guar Gum is almost an order of magnitude smaller at 250 Å in diameter. If pulled out the length of a Polyox molecule would be 750,000 Å and Guar Gum 8000 Å. The outstretched length of a Polyox molecule is about the order of magnitude of a typical small eddy. What is interesting is that although a 150 p.p.m. solution of Guar Gum and a 50 p.p.m. Polyox solution have approximately the same pressure drop reduction characteristics in turbulent pipe flow, they do not have the same properties in free turbulent flows. The reason may be due to the difference in the fine structure of these flows.

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